Exercise 14

Differentiate.

$$y = \frac{\sqrt{x}}{2+x}$$

Solution

Use the quotient rule to differentiate y.

$$\begin{aligned}
y' &= \frac{d}{dx}\left(\frac{x^{1/2}}{2+x}\right) \\
&= \frac{\left[\frac{d}{dx}(x^{1/2})\right](2+x) - \left[\frac{d}{dx}(2+x)\right](x^{1/2})}{(2+x)^2} \\
&= \frac{\left(\frac{1}{2}x^{-1/2}\right)(2+x) - (1)(x^{1/2})}{(2+x)^2} \\
&= \frac{\left(x^{-1/2} + \frac{1}{2}x^{1/2}\right) - (x^{1/2})}{(2+x)^2} \\
&= \frac{x^{-1/2} - \frac{1}{2}x^{1/2}}{(2+x)^2} \cdot \frac{2x^{1/2}}{2x^{1/2}} \\
&= \frac{2-x}{2\sqrt{x}(2+x)^2}
\end{aligned}$$